

$$\sin^6 x + \cos^6 x = \sin x \cos x$$

$$(\sin^2 x)^3 + (\cos^2 x)^3 = \sin x \cos x$$

$$(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) = \sin x \cos x$$

$$\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x = \sin x \cos x$$

$$\sin^4 x - 3 \sin^2 x \cos^2 x + \cos^4 x + 2(\sin^2 x \cos^2 x) = \sin x \cos x$$

$$(\sin^2 + \cos^2) - 3 \sin^2 x \cos^2 x = \sin x \cos x$$

$$1 - 3 \sin^2 x \cos^2 x = \sin x \cos x$$

$$\sin x \cos x = t$$

$$3t^2 + t - 1 = 0$$

$$D = 1 + 12 = 13$$

$$t = \frac{-1 \pm \sqrt{13}}{6}$$

$$\sin x \cos x = \frac{-1 \pm \sqrt{13}}{6}$$

$$\sin 2x = \frac{-1 \pm \sqrt{13}}{3}$$

$$\sin 2x = \frac{-1 - \sqrt{13}}{3} \text{ - не подходит}$$

$$\sin 2x = \frac{-1 + \sqrt{13}}{3}$$

$$2x = \arcsin\left(\frac{-1 + \sqrt{13}}{3}\right) + 2Pk$$

$$2x = \pi - \arcsin\left(\frac{-1 + \sqrt{13}}{3}\right) + 2Pk$$

$$x = \frac{\arcsin\left(\frac{-1 + \sqrt{13}}{3}\right)}{2} + Pk$$

$$x = \frac{\pi}{2} - \frac{\arcsin\left(\frac{-1 + \sqrt{13}}{3}\right)}{2} + Pk$$

$$\sin^5 x + \cos^5 x = 1$$

$$\sin^5 x + \cos^5 x = \sin^2 x + \cos^2 x$$

$$\sin^5 x - \sin^2 x = \cos^2 x - \cos^5 x$$

$$\sin^2 x(\sin^3 x - 1) = \cos^2 x(1 - \cos^3 x)$$

$$\sin^2 x(\sin x - 1)(1 + \sin x + \sin^2 x) = \cos^2 x(1 - \cos x)(1 + \cos x + \cos^2 x)$$

$$[1 - \cos^2 x](\sin x - 1)(1 + \sin x + \sin^2 x) = [1 - \sin^2 x](1 - \cos x)(1 + \cos x + \cos^2 x)$$

$$1 - \cos x = 0$$

$$\cos x = 1$$

$$x = 2Pk$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

$$x = \frac{\pi}{2} + 2Pk$$

$$(1 + \cos x)(\sin^2 x + \sin x + 1) = -(1 + \sin x)(\cos^2 x + \cos x + 1)$$

$$(1 + \cos x)(\sin^2 x + \sin x + 1) + (1 + \sin x)(\cos^2 x + \cos x + 1) = 0$$

$$y(x) = \sin^2 x + \sin x + 1 \text{ -- всегда положительная}$$

$$\sin x = t$$

$$y(t) = t^2 + t + 1$$

$$D = 1 - 4 = -3$$

$$y(x) = \cos^2 x + \cos x + 1 \text{ --- всегда положительная}$$

$$(1 + \cos x)(\sin^2 x + \sin x + 1) + (1 + \sin x)(\cos^2 x + \cos x + 1) = 0$$

$$1 + \sin x \text{ и } 1 + \cos x \text{ не равны } 0 \text{ одновременно}$$

$$\text{Ответ: } 2Pk, \frac{\pi}{2} + 2Pk$$